Workshop Solutions to Sections 2.3 and 2.4

1) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f+g)(x) =$ Solution: $(f+g)(x) = x^2 + \sqrt{4-x}$	2) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f+g} =$ Solution: $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \iff x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{f+g} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
3) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f-g)(x) =$ Solution: $(f-g)(x) = x^2 - \sqrt{4-x}$	4) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f-g} =$ Solution: $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \iff x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{f-g} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
5) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(fg)(x) =$ Solution: $(fg)(x) = x^2\sqrt{4-x}$	6) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{fg} =$ Solution: $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \iff x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{fg} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
7) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f \circ g)(x) =$ Solution: $(f \circ g)(x) = f(g(x))$ $= f(\sqrt{4-x}) = (\sqrt{4-x})^2 = 4 - x$	8) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f \circ g} =$ Solution: $(f \circ g)(x) = f(g(x))$ $= f(\sqrt{4-x}) = (\sqrt{4-x})^2 = 4 - x$ $D_g = (-\infty, 4]$ $D_{f(g(x))} = \mathbb{R}$ $D_{f \circ g} = D_g \cap D_{f(g(x))} = (-\infty, 4] \cap \mathbb{R} = (-\infty, 4]$
9) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(g \circ f)(x) =$ Solution: $(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{4-x^2}$	10) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{g \circ f} =$ Solution: $(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{4-x^2}$ $D_f = \mathbb{R}$ $D_{g(f(x))} = [-2,2]$ $D_{g \circ f} = D_f \cap D_{g(f(x))} = \mathbb{R} \cap [-2,2] = [-2,2]$
11) If $f(x) = x^2$, then $(f \circ f)(x) =$ Solution: $(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$	12) If $f(x) = x^2$, then $D_{f \circ f} =$ Solution: $(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$ $D_f = \mathbb{R}$ $D_{f(f(x))} = \mathbb{R}$ $D_{f \circ f} = D_f \cap D_{f(f(x))} = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

13) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{f}{g}\right)(x) =$

Solution:

$$\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$$

14) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\frac{f}{g}} =$

Solution:

$$\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$$

$D_f = \mathbb{R}$

$g(x)$ is defined when $4 - x \geq 0 \iff x \leq 4$. Thus, $D_g = (-\infty, 4]$

$$D_{\frac{f}{g}} = \{x \in D_f \cap D_g | g(x) \neq 0\}$$
$$= \mathbb{R} \cap (-\infty, 4) = (-\infty, 4)$$

15) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{g}{f}\right)(x) =$

Solution:

$$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$$

16) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\frac{g}{f}} =$

Solution:

$$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$$

$D_f = \mathbb{R}$

$g(x)$ is defined when $4 - x \geq 0 \iff x \leq 4$. Thus, $D_g = (-\infty, 4]$

$$D_{\frac{g}{f}} = \{x \in D_f \cap D_g | f(x) \neq 0\}$$
$$= \mathbb{R} \setminus \{0\} \cap (-\infty, 4] = (-\infty, 0) \cup (0, 4]$$

17) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f+g)(x) =$

Solution:

$$(f+g)(x) = (9 - x^2) + (10) = 9 - x^2 + 10$$
$$= 19 - x^2$$

18) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f-g)(x) =$

Solution:

$$(f-g)(x) = (9 - x^2) - (10) = 9 - x^2 - 10$$
$$= -x^2 - 1$$

19) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g-f)(x) =$

Solution:

$$(g-f)(x) = (10) - (9 - x^2) = 10 - 9 + x^2$$
$$= 1 + x^2$$

20) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(fg)(x) =$

Solution:

$$(fg)(x) = (9 - x^2)(10) = 90 - 10x^2$$

21) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f \circ g)(x) =$

Solution:

$$(f \circ g)(x) = f(g(x)) = f(10)$$
$$= 9 - 10^2 = 9 - 100 = -91$$

22) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g \circ f)(x) =$

Solution:

$$(g \circ f)(x) = g(f(x)) = g(9 - x^2) = 10$$

23) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f \circ f)(x) =$

Solution:

$$(f \circ f)(x) = f(f(x)) = f(9 - x^2)$$
$$= 9 - (9 - x^2)^2$$

24) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g \circ g)(x) =$

Solution:

$$(g \circ g)(x) = g(g(x)) = g(10) = 10$$

25) If $f(x) = 9 - x^2$, $g(x) = \sin x$ and $h(x) = 3x + 2$, then $(f \circ g \circ h)(x) =$

Solution:

$$(f \circ g \circ h)(x) = f\left(g(h(x))\right)$$
$$= f\left(g(3x+2)\right)$$
$$= f(\sin(3x+2))$$
$$= 9 - (\sin(3x+2))^2$$
$$= 9 - sin^2(3x+2)$$

26) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then $(f+g)(x) =$

Solution:

$$(f+g)(x) = \sqrt{25 + x^2} + x^3$$

27) If $f(x) = \sqrt{25+x^2}$ and $g(x) = x^3$, then $(f-g)(x) =$ Solution: $(f-g)(x) = \sqrt{25+x^2} - x^3$	28) If $f(x) = \sqrt{25+x^2}$ and $g(x) = x^3$, then $(fg)(x) =$ Solution: $(fg)(x) = x^3\sqrt{25+x^2}$
29) If $f(x) = \sqrt{25+x^2}$ and $g(x) = x^3$, then $\left(\frac{f}{g}\right)(x) =$ Solution: $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{25+x^2}}{x^3}$	30) If $f(x) = \sqrt{25+x^2}$ and $g(x) = x^3$, then $(f \circ g)(x) =$ Solution: $(f \circ g)(x) = f(g(x)) = f(x^3) = \sqrt{25+(x^3)^2}$ $= \sqrt{25+x^6}$
31) If $f(x) = \sqrt{25+x^2}$ and $g(x) = x^3$, then $(g \circ f)(x) =$ Solution: $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{25+x^2}\right) = \left(\sqrt{25+x^2}\right)^3$ $= \sqrt{(25+x^2)^3}$	32) If $f(x) = \sqrt{x}$ and $g(x) = x-2$, then $(f \circ g)(x) =$ Solution: $(f \circ g)(x) = f(g(x)) = f(x-2) = \sqrt{x-2}$
33) If $f(x) = \sqrt{x}$ and $g(x) = x-2$, then $(g \circ f)(x) =$ Solution: $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{x} - 2$	34) If $f(x) = \sqrt{x}$ and $g(x) = x-2$, then $(g \circ g)(x) =$ Solution: $(g \circ g)(x) = g(g(x)) = g(x-2) = (x-2)-2$ $= x-2-2 = x-4$
35) If $f(x) = \sqrt{x}$ and $g(x) = x-2$, then $(fg)(x) =$ Solution: $(fg)(x) = (\sqrt{x})(x-2) = (x-2)\sqrt{x}$	36) If $f(x) = \sin 5x$ and $g(x) = x^2+3$, then $(f \circ g)(x) =$ Solution: $(f \circ g)(x) = f(g(x)) = f(x^2+3) = \sin 5(x^2+3)$
37) If $f(x) = \sin 5x$ and $g(x) = x^2+3$, then $(g \circ f)(x) =$ Solution: $(g \circ f)(x) = g(f(x)) = g(\sin 5x) = (\sin 5x)^2 + 3$ $= sin^2 5x + 3$	38) If $f(x) = \sin 5x$ and $g(x) = x^2+3$, then $(fg)(x) =$ Solution: $(fg)(x) = (\sin 5x)(x^2+3) = (x^2+3)\sin 5x$
39) If $f(x) = \sqrt{x}$ and $g(x) = \cos x$, then $(g \circ f)(x) =$ Solution: $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \cos\sqrt{x}$	40) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1-x^2$, then $(f \circ g)(x) =$ Solution: $(f \circ g)(x) = f(g(x)) = f(1-x^2) = (1-x^2) + \frac{1}{1-x^2}$
41) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1-x^2$, then $(g \circ f)(x) =$ Solution: $(g \circ f)(x) = g(f(x)) = g\left(x+\frac{1}{x}\right) = 1 - \left(x+\frac{1}{x}\right)^2$	42) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1-x^2$, then $(fg)(x) =$ Solution: $(fg)(x) = \left(x+\frac{1}{x}\right)(1-x^2)$
43) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units upwards , then the new graph represented the graph of the function is Solution: x^2+2	44) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units downwards , then the new graph represented the graph of the function is Solution: x^2-2
45) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units to the right , then the new graph represented the graph of the function is Solution: $(x-2)^2 = x^2-4x+4$	46) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units to the left , then the new graph represented the graph of the function is Solution: $(x+2)^2 = x^2+4x+4$