Analytical Expressions for the Physical Characteristics Near Interior Points of Polytropes

M.A. Sharaf[1] and .A. N. Al-Amoudi[2]

[1]Department of Astronomy, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia
[2]Department of Physics,Faculty of Science,King Abdul Aziz University,Jeddah,Saudi Arabia.
Email:Sharaf_adel@hotmail.com

Abstract. In this paper, literal analytical expressions in power series forms are developed for the physical characteristics near interior points of polytropes**.**

Keywords **:** Stellar interior **,** polytropes

1.Introduction

Polytropic models are vital for two classes of theoretical astrophysics: stellar structure [1] and galactic dynamics[2,3]. In stellar structure one can obtain the march of the physical variables inside polytropic equilibrium configurations [4]. Most stellar models had more or less direct bearing on polytropes, and many analytical solutions to the internal constitution of stars are connected with polytropic relationships.

As, for examples, convective stellar models with radiation pressures and without radiation pressures were considered in full details long time ago [5,6] have estimated the polytropic indices of planetary models obeying the polytropic equation of state. Magnetic braking of the rotation of pre-main sequence stars approximated as fully convective $n = 1.5$ polytropes ,has been calculated by [7,8,] Vandervoort[9] has considered an application to galactic bars of nonaxisymmetric ,triaxial polytropes with index $0.5 \leq n \leq 0.808$.

Moreover, particular models give systematic expositions of theoretical results, which bear some interest on the structure of rotating stars [10]. White dwarfs and neutron stars are best studied using differentially rotating polytropic cores [11] Most stellar models had more or less direct bearing on polytropes, and many analytical solutions to the internal constitution of stars are connected with polytropic relationships . A model of pulsar that has a magnetized core was presented [12] and the behavior in evolution of spin and luminosity was analyzed. The stars are supposed to be polytropes, thereby rendering all physical quantities calculable with any desired precision. Recently Gonzalo [13] has investigated spherically symmetric, static matter configurations with polytropic equation of state for a class of f (R) models in Palatini Formalism.

In fact, the polytrope representation of stars   models, is a method that today still lends valuable technique and insights to the internal structure of stars .It is also proven to be most versatile in examination of a variety of situations ,including the analysis of isothermal cores, convective stellar   interiors   and   fully   degenerate   stellar   configurations. Moreover,composite polytropes are used to construct spherical ,hydrostatic models of molecular clouds[14].Recently Cristina [15] obtained in the presence of a weak poloidal magnetic filed, the filed distribution in a sequence of composite polytrobic stars with uniform density throughout the core of polytropic index n=0, and an envelope with n=1.

On the other hand, in galactic dynamics, the Lane-Emden equation of the polytropic equilibrium configurations is considered as generating function of potential models for flattened galactic systems, upon such potentials the forces and the star orbits in these systems could be determined. These latter results are extremely important in galactic

studies; one of their most explored aspects is the problem of the correlation between the parameters of the orbit of a star and its physical properties. Recently Tuns *et al* [16] used initial configurations with n=3 polytropes for axisymmetric simulations of the, magnetorotational collapse of very massive stars. Moreover, the effects of a positive cosmological constant on astrophysical and cosmological configurations were described by the polytrobic equation of state [17].

From the above brief notes, one can detected that, why a great effort has been devoted up to now, and is being devoted at present to express some characteristic physical parameters related to ploytrobic models [18-20].

The basic equation for ploytrobic configurations is the Lane-Emden differential equation.

In fact, in the absence of closed analytical solution of a given differential equation the power series solution (which of course assumed to be convergent) can serve as the analytical representation of its solution. Moreover, it is worth noting that the power series is one of the most powerful methods of mathematical analysis and is no less (and some – times even more) convenient than the elementary functions especially when the problems are to be studied on computers. In fact, most computers often use series in the calculations of the majority of the elementary functions . The analytical solution for this equation was established [21] in a form of power series, which can be convergent for any allowable indices n. An accelerated form of these power series are developed by Saad [22] using Pade' technique and changing of the independent variable .For polytropic and isothermal gas spheres (N=3),Nouh [23] improved the convergence of the power

series by using a combination of two accelerating techniques Eular-Abel transformation and Pade' approximation.

The physical characteristics of polytropic configurations depend in turn on the solutions of Lane-Emden equation and have been widely quoted in the astrophysical literatures.

Due to the importance of ploytrobic configurations, as mentioned in brief, and the needs of having numerical and analytical treatments for their structures so as to suit any application are what motivated the present paper with the following objective:

"To establish literal analytical expressions in power series forms for the physical characteristics of the ploytrobic configurations."

The importance of these expressions is due to some factors, Smoe of these are:(i) their analytical forms, offer in general much deeper insight into the nature of the physical characteristics to which they refer. (ii)These expressions are general in the sense that they could be used for any polytrobic index n, so that they can suit many of the applications. (iii)The coefficients of each of these expressions have been found directly without the use of recurrence formulae.

2. Lane - Emden Equation and Its Analytical Solution

Consider equations of state of the form

$$P = K \rho^{\gamma} \quad , \tag{1}$$

where   K and γ are constants ,known as polytropic equations of state. It is customary to define the corresponding polytropic index ,denoted by n ,as

$$\gamma = 1 + \frac{1}{n} \quad . \tag{2}$$

Let r be the space coordinate giving the distance from the center of the star. At the center $r = 0$,and at the surface $r = R$, the radius of the star, then $\rho \equiv \rho(r)$ and $P \equiv P(r)$ are respectively ,the density and the pressure at distance r from the center .

Lane-Emden equation of index n is given by [e.g. Prialnik 2007]

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\theta}{d\xi}\right) = -\theta^n , \qquad (3)$$

where the dimensionless variable ξ, is defined by

$$r = \alpha \xi \qquad ; \qquad \alpha = \left(\frac{(n+1)\,K}{4\pi\,G}\rho_c^{\frac{1}{n}-1}\right)^{1/2}$$

ρ_c, the central density .and G is the gravitational constant with its numerical value equals

$G = 6.673 \times 10^{-8}\,cm^3/gm/sec^2$.

The relation between ρ and θ is given as :

$$\rho = \rho_c\,\theta^n \qquad (4)$$

Equation (3) is to be solved under the conditions:

$$\text{at } \xi = 0 \; ; \; \theta = 1.; \frac{d\theta}{d\xi} = 0. \qquad (5)$$

With these initial conditions ,the second order differential Equation (3) (Lane-Emden equation) will possess a unique solution .This solution is called the Lane-Emden function of index n and is denoted by

$$\theta = \theta_n(\xi) .$$

2.1 *Power Series Solution of Lane-Emden Equation* $\forall\ 0 \le \xi \le 1$

It could be shown that, the power series solution of Lane-Emden
equation valid $\forall\ 0 \le \xi \le 1$ is

$$\theta_n(\xi) = 1 + \sum_{k=1}^{\infty} a_k \xi^{2k}\ , \qquad\qquad (6)$$

$$a_1 = -\frac{1}{6} \qquad\qquad (7)$$

which satisfies the initial conditions of Equation (5) , and the a's
coefficients are completely determined in a full recursive way from
Equation (7) and

$$a_{k+1} = \frac{1}{k(k+1)(2k+3)} \sum_{i=1}^{k} (ni-k+i)(2k-2i+3)(k+1-i)a_i a_{k+1-i} \qquad \forall\ k \ge 1. \quad (8)$$

The well known closed analytical solution of Lane-Emden Equation
for n = 0,1 and 5 could easily be obtained from Equations (6) and (8)
, and we get

$$\left. \begin{array}{l} \theta_0(\xi) = 1 - \dfrac{1}{6}\xi^2 \\[2ex] \theta_1(\xi) = \dfrac{\sin\xi}{\xi} \\[2ex] \theta_5(\xi) = \left(1 + \dfrac{1}{3}\xi^2\right)^{-1/2} \end{array} \right\} \qquad\qquad (9)$$

2.2 *Symbolic expressions of the a's coefficients*

Due to the limited space, the symbolic expressions of the a's
coefficients are listed in Table A1 for $j = 1,2,...7$

TABLE A1: Symbolic expressions of the coefficients $a_j ; j = 1,2,...7$

$$a_1 = -\frac{1}{6}$$

$$a_2 = \frac{n}{120}$$

$$a_3 = \frac{(5-8n)\,n}{15120}$$

$$a_4 = \frac{n\,(70 + 61\,n\,(-3 + 2\,n))}{3265920}$$

$$a_5 = -\frac{n\,(-3150 + n\,(10805 + 2\,n\,(-6321 + 2516\,n)))}{1796256000}$$

$$a_6 = \frac{n\,(138600 + n\,(-574850 + n\,(915935 + 2\,n\,(-331583 + 91808\,n))))}{840647808000}$$

$$a_7 = -\frac{n\,(-21021000 + n\,(101038350 + n\,(-199037015 + 2\,n\,(100286893 + 4\,n\,(-12897299 + 2703152\,n)))))}{1235752277760000}$$

3. Symbolic Expressions of the Physical Characteristics of Polytropes

In this section literal analytical expressions in power series forms are established for the physical characteristics of polytropes .These series converge very rabidly $\forall 0 \leq \xi \leq 1$. Due to the limited space , only the first five symbolic expressions of the coefficients for each physical characteristics will be listed in a separate table.

3.1 The Mass

The mass interior to ξ is given as:

$$M(\xi) = 4\,\pi\alpha^3\,\rho_c\left(-\xi^2\,\frac{d\theta}{d\xi}\right)$$

then using Equation (6) we get

$$M(\xi) = \sum_{j=1}^{\infty}\lambda_j\,\xi^{2j+1} \quad ,$$

$$\lambda_j = -8\pi j\alpha^3\rho_c a_j$$

Symbolic expressions of the coefficients $\lambda_j ; j = 1,2,3,4,5$ are listed in Table A2

TABLE A2: **Symbolic expressions of the coefficients** $\lambda_j; j = 1,2,3,4,5$

$$\lambda_1 = \frac{4}{3} \pi \alpha^3 \rho_c$$

$$\lambda_2 = -\frac{2}{15} n \pi \alpha^3 \rho_c$$

$$\lambda_3 = \frac{1}{630} n (-5 + 8n) \pi \alpha^3 \rho_c$$

$$\lambda_4 = -\frac{n (70 + 61 n (-3 + 2n)) \pi \alpha^3 \rho_c}{102060}$$

$$\lambda_5 = \frac{n (-3150 + n (10805 + 2n (-6321 + 2516 n))) \pi \alpha^3 \rho_c}{44906400}$$

3.2 The Mass-radius relation

The mass-radius relation is given as

$$\frac{M(\xi)}{r(\xi)} = \frac{4 \pi \alpha^3 \rho_c \left(-\xi^2 \frac{d\theta}{d\xi}\right)}{\alpha \xi} \Rightarrow \frac{M(\xi)}{r(\xi)} = -4 \pi \alpha^2 \rho_c \xi \frac{d\theta}{d\xi}$$

Using Equation (6) we get

$$\frac{M(\xi)}{r(\xi)} = \sum_{j=1}^{\infty} \nu_j \xi^{2j} \quad ,$$

$$\nu_j = -8 \pi \alpha^2 \rho_c \, j a_j.$$

Symbolic expressions of the coefficients $\nu_j; j = 1,2,3,4,5$ are listed in

Table A3

TABLE A3: Symbolic expressions of the coefficients $\nu_j; j = 1,2,3,4,5$

$$\nu_1 = \frac{4}{3}\,\pi\,\alpha^2\,\rho_c$$

$$\nu_2 = -\frac{2}{15}\,n\,\pi\,\alpha^2\,\rho_c$$

$$\nu_3 = \frac{1}{630}\,n\,(-5 + 8\,n)\,\pi\,\alpha^2\,\rho_c$$

$$\nu_4 = -\frac{n\,(70 + 61\,n\,(-3 + 2\,n))\,\pi\,\alpha^2\,\rho_c}{102060}$$

$$\nu_5 = \frac{n\,(-3150 + n\,(10805 + 2\,n\,(-6321 + 2516\,n)))\,\pi\,\alpha^2\,\rho_c}{44906400}$$

3.3 Central condensation

The central condensation is given as

$$\frac{\rho_c}{\rho} = -\frac{3}{\xi}\left(\frac{d\theta}{d\xi}\right)^{-1}.$$

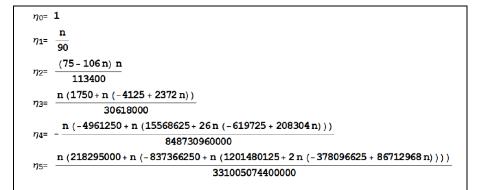where $\overline{\rho}$ is the mean density. Using Equation (6) we get

$$\frac{\rho_c}{\overline{\rho}} = \sum_{j=0}^{\infty}\eta_j\xi^{2j},$$

$$\eta_0 = 1,$$

$$\eta_m = \frac{2}{3}\sum_{k=1}^{m}(k+1)\,a_{k+1}\eta_{m-k} \quad \forall m \geq 1.$$

Symbolic expressions of the coefficients $\eta_j; j = 0,1,2,3,4,5$ are listed in

Table A4

TABLE A4: **Symbolic expressions of the coefficients** $\eta_j; j = 0,1,2,3,4,5$

$$\eta_0 = 1$$

$$\eta_1 = \frac{n}{90}$$

$$\eta_2 = \frac{(75 - 106\,n)\,n}{113400}$$

$$\eta_3 = \frac{n\,(1750 + n\,(-4125 + 2372\,n))}{30618000}$$

$$\eta_4 = -\frac{n\,(-4961250 + n\,(15568625 + 26\,n\,(-619725 + 208304\,n)))}{848730960000}$$

$$\eta_5 = \frac{n\,(218295000 + n\,(-837366250 + n\,(1201480125 + 2\,n\,(-378096625 + 86712968\,n))))}{331005074400000}$$

3.4 The Temperature

The temperature is given as

$$T = T_c \theta \ .$$

Using Equation (6),we get

$$T = \sum_{j=0}^{\infty} \gamma_j \xi^{2j}, \ \gamma_0 = T_c, \ \gamma_j = a_j T_c \ .$$

Symbolic expressions of the coefficients $\gamma_j; j = 0,1,2,3,4,5$ are listed in Table A5

TABLE A5: Symbolic expressions of the coefficients $\gamma_j; j = 0,1,2,3,4,5$

$$\gamma_0 = T_c$$

$$\gamma_1 = -\frac{T_c}{6}$$

$$\gamma_2 = \frac{n\, T_c}{120}$$

$$\gamma_3 = -\frac{n\,(-5 + 8\,n)\, T_c}{15120}$$

$$\gamma_4 = \frac{n\,(70 + 61\,n\,(-3 + 2\,n))\, T_c}{3265920}$$

$$\gamma_5 = -\frac{n\,(-3150 + n\,(10805 + 2\,n\,(-6321 + 2516\,n)))\, T_c}{1796256000}$$

3.5 Pressure

The pressure is given as

$$P(\xi) = P_c\,\theta^{n+1}.$$

Using Equation (6), we get

$$P(\xi) = \sum_{j=0}^{\infty} \beta_j \xi^{2j},$$

$$\beta_0 = P_c,$$

$$\beta_m = \frac{1}{m}\sum_{k=1}^{m}(nk + 2k - m)a_k\beta_{m-k} \quad \forall m \geq 1.$$

Symbolic expressions of the coefficients $\beta_j; j = 0,1,2,3,4,5$ are listed in Table A6

TABLE A6: Symbolic expressions of the coefficients $\beta_j; j = 0,1,2,3,4,5$

$$\beta_0 = P_c$$

$$\beta_1 = -\frac{1}{6}(1+n)P_c$$

$$\beta_2 = \frac{1}{45}n(1+n)P_c$$

$$\beta_3 = -\frac{n(1+n)(-25+61n)P_c}{22680}$$

$$\beta_4 = \frac{n(1+n)(175+n(-660+629n))P_c}{2041200}$$

$$\beta_5 = -\frac{n(1+n)(-11025+n(52310+n(-84309+45904n)))P_c}{1347192000}$$

3.6 The Density

Since

$$\rho = \rho_c \theta^n.$$

Using Equation(6) we get

$$\rho = \sum_{j=0}^{\infty} \chi_j \xi^{2j},$$

$$\chi_0 = \rho_c,$$

$$\chi_m = \frac{1}{m}\sum_{k=1}^{m}(kn-m+k)a_k\chi_{m-k}; \quad \forall m \geq 1.$$

Symbolic expressions of the coefficients $\chi_j; j = 0,1,2,3,4,5$ are listed in Table A7

TABLE A7: Symbolic expressions of the coefficients $\chi_j; j = 0,1,2,3,4,5$

$$\chi_0 = \rho_c$$

$$\chi_1 = -\frac{n \rho_c}{6}$$

$$\chi_2 = \frac{1}{360} n (-5 + 8 n) \rho_c$$

$$\chi_3 = -\frac{n (70 + 61 n (-3 + 2 n)) \rho_c}{45360}$$

$$\chi_4 = \frac{n (-3150 + n (10805 + 2 n (-6321 + 2516 n))) \rho_c}{16329600}$$

$$\chi_5 = -\frac{n (138600 + n (-574850 + n (915935 + 2 n (-331583 + 91808 n)))) \rho_c}{5388768000}$$

3.7 Gravity

The gravity $g(\xi)$ could be written as

$$g(\xi) = -4\pi\, G \left\{ \frac{(n+1)\, K}{4\pi\, G} \right\}^{1/2} \rho_c^{\frac{n+1}{2n}} \frac{d\theta}{d\xi} \; .$$

Then using Equation (6) we get

$$g(\xi) = \sum_{j=1}^{\infty} \delta_j \xi^{2j-1} ,$$

$$\delta_j = -8\,\pi\, j\, G \left\{ \frac{(n+1)\, K}{4\pi\, G} \right\}^{1/2} \rho_c^{\frac{n+1}{2n}} a_j \; .$$

Symbolic expressions   of the coefficients $\delta_j; j = 1,2,3,4,5$ are listed in Table A8
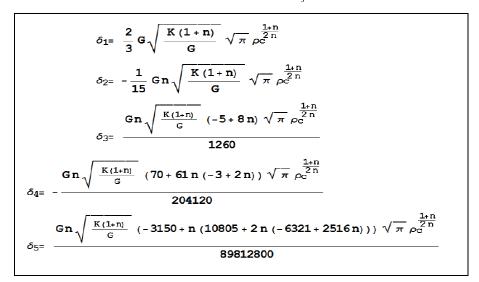
TABLE A8: Symbolic expressions of the coefficients $\delta_j; j = 1,2,3,4,5$

$$\delta_1 = \frac{2}{3} G \sqrt{\frac{K(1+n)}{G}} \sqrt{\pi}\ \rho_c^{\frac{1+n}{2n}}$$

$$\delta_2 = -\frac{1}{15} Gn \sqrt{\frac{K(1+n)}{G}} \sqrt{\pi}\ \rho_c^{\frac{1+n}{2n}}$$

$$\delta_3 = \frac{Gn \sqrt{\frac{K(1+n)}{G}}\ (-5+8n)\ \sqrt{\pi}\ \rho_c^{\frac{1+n}{2n}}}{1260}$$

$$\delta_4 = -\frac{Gn \sqrt{\frac{K(1+n)}{G}}\ (70+61n(-3+2n))\ \sqrt{\pi}\ \rho_c^{\frac{1+n}{2n}}}{204120}$$

$$\delta_5 = \frac{Gn \sqrt{\frac{K(1+n)}{G}}\ (-3150+n(10805+2n(-6321+2516n)))\ \sqrt{\pi}\ \rho_c^{\frac{1+n}{2n}}}{89812800}$$

3.8 Gravitational acceleration

The gravitational acceleration is given by

$$F = -G\frac{M(\xi)}{r^2(\xi)} = -G\frac{4\pi\alpha^3\rho_c\left(-\xi^2\frac{d\theta}{d\xi}\right)}{\alpha^2\xi^2} = 4\pi\alpha G\rho_c\frac{d\theta}{d\xi}.$$
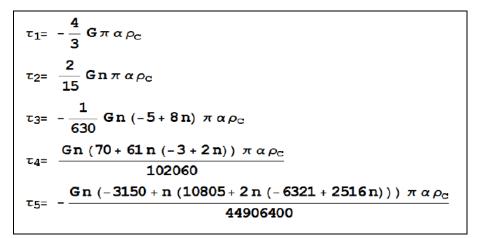
 Using Equation (6), we get

$$F = \sum_{j=1}^{\infty}\tau_j\xi^{2j-1}\ ,$$

$$\tau_j = 8 \pi \alpha \rho_c G \, j a_j.$$

Symbolic expressions of the coefficients $\tau_j; j = 1,2,3,4,5$ are listed in Table A9

TABLE A9: **Symbolic expressions of the coefficients** $\tau_j; j = 1,2,3,4,5$

$$\tau_1 = -\frac{4}{3} G \pi \alpha \rho_c$$

$$\tau_2 = \frac{2}{15} G n \pi \alpha \rho_c$$

$$\tau_3 = -\frac{1}{630} G n (-5 + 8n) \pi \alpha \rho_c$$

$$\tau_4 = \frac{G n (70 + 61 n (-3 + 2n)) \pi \alpha \rho_c}{102060}$$

$$\tau_5 = -\frac{G n (-3150 + n (10805 + 2 n (-6321 + 2516n))) \pi \alpha \rho_c}{44906400}$$

Conclusion

In concluding the present paper, literal analytical expressions in power series forms are developed for the physical characteristics near interior points of polytropes
The importance of these expressions are due to some facts ,of these are the following

1-The expressions are obtained as power series in ξ .Consequently ,we can obtain physical characteristics X (say) very simply and efficiently by using any power series evaluation algorithms . On the other hand ,the solution of Lane-Emden equation to obtain the physical characteristics X by any numerical differential equation solver gives us X only at *definite* values of ξ belonging to the set S, where

$$S = \{0, \ \Delta\xi, \ 2\Delta\xi, \ 3\Delta\xi, \cdots \ \}$$

where $\Delta\xi$ the step size used in numerical differential equation solver. So if we need the values of X at $\xi^* \notin S$, we must apply an interpolation formula. A process, which needs more execution time, moreover, which is the most critical, the loss of accuracy that usually associated with the usage of interpolation formula.

2-The analytical power series representations of the physical characteristics are *invariant* under many operations because, addition, multiplication, exponent ion, integration, different ion, etc of a power series is also a power series .A fact which provides excellent flexibility in dealing with analytical as well as computational developments of the problems of polytropes

Once more , numerical differential equation solver can not ,by any way , provide such flexibility. Moreover, these analytical formulae usually offer much deeper insight into the nature of a physical characteristic as compared to numerical integration.

References

[1] **Prialnik,D.**,An Indroduction to the theory of Stellar Structure and Evolution, *Cambridge University Press,* (2007).

[2] **Binney, J. and Tremains, S.**, Galactic Dynamics, *Princeton Univ. Press, Princeton, New Jersey*, (1987)*.*

[3] **Bertin ,G.**,Dynamics of Galaxies**,** *Cambridge University Press* , 2000.

[4] **Hansen,C.J.,Kawaler, S.D,and Trimble,V**.,Stellar Interiors*: Physical principles,structure,and evolution-*[2nd] ed**.,** *Springer,New York ,* (2004).

[5] *Öpik* **,E.J**.,Jupiter:Chemical composition,structure,and origin of a giant planet.*Icarus* **1**: 200-257, (1962).

[6] **Bobrow,A.M.,Vasil'ev,P.P.,Zharkov,V.N.,Trubitsyn,V.P**.,The equation of hydrostatic equilibrium in the theory of a planetary figure, ,*Astron.Zh*.**55**: 857-861,(1978).

[7] **Okamoto,I.**,On the loss of angular momentum from the protosun and the formation of the solar system, *Publ.Astron.Soc.Japan* **21**: 25-53, (1969).

[8] Okamoto,I.,On the loss of angular momentum from the stars in the pre-sequence phase, In Stellar Rotation. Edited by A.Slettebak.D.Reidel Publ.,*Dordrecht .* **73**-81, (1970).

[9] **Vandervoort,P.O**.,The equilibrium of a galactic bar,*Astrophys.J*.**240**,478-487 *AJSE,King Fahd,Univ*,**29 (1A)** : 67-82, (1980).

[10] **Tassoul, J. L**, , Theory of Rotating Stars, *Princeton Univ. Press, Princeton, New Jersey*, (1978).

[11] **Eriguch,Y.andMliller,E**., Equilibrium model of differentially rotating polytropes and the collapse of rotating stellar cores,*Astron.Astrophys.***147**,161-168, 1985

[12] **Koichi,T**., Polytropic Model of Pulsars with a Strongly Magnetized Core and Their Spin Evolutions ,*Publ.Astron Soc.Jpn*,**57**(2):365-374, (2005).

[13] **Gonzalo ,O,J**., Reexamination of Polytropic Spheres in Palatini f(R) gravity,*Phsical Review* **D**,78, (2008).

[14] **Charles,C.L.,andChristopher,F.,**Composite polytrope models for molecular clouds,*ApJ*,**528**: (2000).

[15] **Cristina,B.**, Composite magnetic polytropes, Fifty Years of Romanian Astrophysics .*AIP Conference Proceeding,* Volume **895**: 234 (2007).

[16] **Tung,L.Y., Shapiro,S.L**., and **Branson,S.C**., magnetorotational collapse of very massive stars to black holes in full general relativity, *Physical Review* **D ,76**:(8) (2007).

[17]**Antolinez,A.B.,Mota,D.F.,andNowakowski,M**., Astrophsical configurations with background cosmology probing dark energy at astrophysical scales,*MNRS,***382**(2),621, (2007).

[18] **Sharaf, M. A., El-Zafarani. H. M; Sharaf, A. A. ,Shaltout, M. A.** and **Hamid, R. H.**, Mass Distribution Near Interior Points of N-dimensional Radilly Symmetric Polytropes, Proceeding of the SOHO6 / GONG 98 Workshop, '*Structure and Dynamics of the Interior of the Sun and Sun like stars', Boston, USA* , (1998).

[19] **Sharaf,M.A.,Almleaky,Y.M,Malawi,A.A,Goharji,A.A** and **Basurah, H.M** , SymbolicAnalytical Expressions of the Physical Characteristics for N-Dimensional Radially Symmetrical Isothermal Models, (2004).

[20] **Horedt. G. P**, ,Polytropes : Apllications in Astrophysics and Related Fields, *Kluwer Academic Publishers*, (2004).

[21] **Sharaf, A. A.** and **Sharaf, M. A**., Power Series Solutions of the Lane-Emden Equation, *Bull. Fac. Sci., Cairo Univ*., **62**:121, (1994).

[22] **Saad,A.A**., Symbolic analytical solution of N-dimensional radially symmetric polytropes,*Astron.Nachr,***325**(9):733, (2004).

[23] **Nouh ,M.I**., Accelerated power series solution of polytropic and isothermal gas spheres , *New Astronomy,* **9**:467, (2004).

تعبيرات تحليلية للخواص الفيزيائية بالقرب من النقاط الداخلية للبولى تروب

محمد عادل شرف[1] ، و العنود عبد الله العمودى[2]

1– قسم العلوم الفلكية – كلية العلوم – جامعة الملك عبد العزيز

2– قسم الفيزياء – كلية العلوم – جامعة الملك عبد العزيز

Sharaf_adel@hotmail.com

المستخلص. في هذا البحث محاولة لعمل قياسات تحليلية بإستخدام متسلسلة أسية للخواص الفيزيائية بالقرب من النقاط الداخلية للبولى تروب.

كلمات مفتاحية: باطن النجوم , بولى تروب