Example 3.10

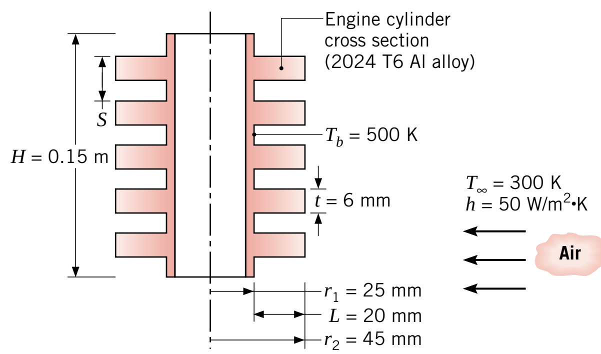
The engine cylinder of a motorcycle is constructed of 2024-T6 aluminum alloy and is of height $H = 0.15$ m and outside diameter $D = 50$ mm. Under typical operating conditions the outer surface of the cylinder is at a temperature of 500 K and is exposed to ambient air at 300 K, with a convection coefficient of 50 W/m$^2 \cdot$ K. Annular fins are integrally cast with the cylinder to increase heat transfer to the surroundings. Consider five such fins, which are of thickness $t = 6$ mm, length $L = 20$ mm, and equally spaced. What is the increase in heat transfer due to use of the fins?

Solution

Known: Operating conditions of a finned motorcycle cylinder.

Find: Increase in heat transfer associated with using fins.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional radial conduction in fins.
3. Constant properties.
4. Negligible radiation exchange with surroundings.
5. Uniform convection coefficient over outer surface (with or without fins).

Properties: Table A.1, 2024-T6 aluminum ($T = 400$ K): $k = 186$ W/m $\cdot$ K.

Analysis: With the fins in place, the heat transfer rate is given by Equation 3.101

$$q_t = hA_t\left[1 - \frac{NA_f}{A_t}(1 - \eta_f)\right]\theta_b$$

where $A_f = 2\pi(r_{2c}^2 - r_1^2) = 2\pi[(0.048\text{ m})^2 - (0.025\text{ m})^2] = 0.0105\text{ m}^2$ and, from Equation 3.99, $A_t = NA_f + 2\pi r_1(H - Nt) = 0.0527\text{ m}^2 + 2\pi(0.025\text{ m})\,[0.15\text{ m} - 0.03\text{ m}] = 0.0716\text{ m}^2$. With $r_{2c}/r_1 = 1.92$, $L_c = 0.023$ m, $A_p = 1.380 \times 10^{-4}\text{ m}^2$, we obtain $L_c^{3/2}(h/kA_p)^{1/2} = 0.15$. Hence, from Figure 3.19, the fin efficiency is $\eta_f \approx 0.95$.

With the fins, the total heat transfer rate is then

$$q_t = 50\ \text{W/m}^2 \cdot \text{K} \times 0.0716\ \text{m}^2 \left[1 - \frac{0.0527\ \text{m}^2}{0.0716\ \text{m}^2}(0.05)\right] 200\ \text{K} = 690\ \text{W}$$

Without the fins, the convection heat transfer rate would be

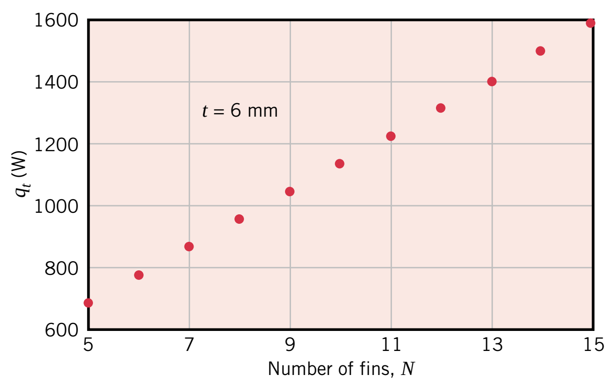
$$q_{\text{wo}} = h(2\pi r_1 H)\theta_b = 50\ \text{W/m}^2 \cdot \text{K}(2\pi \times 0.025\ \text{m} \times 0.15\ \text{m})200\ \text{K} = 236\ \text{W}$$

Hence

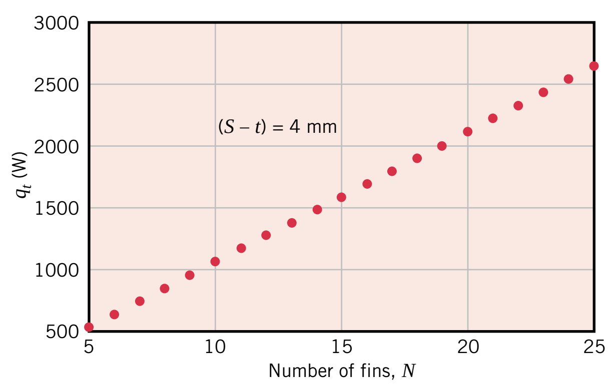
$$\Delta q = q_t - q_{\text{wo}} = 454\ \text{W}$$ ◁

Comments:

1. Although the fins significantly increase heat transfer from the cylinder, considerable improvement could still be obtained by increasing the number of fins. We assess this possibility by computing q_t as a function of N, first by fixing the fin thickness at $t = 6$ mm and increasing the number of fins by reducing the spacing between fins. Prescribing a fin clearance of 2 mm at each end of the array and a minimum fin gap of 4 mm, the maximum allowable number of fins is $N = H/S = 0.15\ \text{m}/(0.004 + 0.006)\ \text{m} = 15$. The parametric calculations yield the following variation of q_t with N:



The number of fins could also be increased by reducing the fin thickness. If the fin gap is fixed at $(S - t) = 4$ mm and manufacturing constraints dictate a minimum allowable fin thickness of 2 mm, up to $N = 25$ fins may be accommodated. In this case the parametric calculations yield

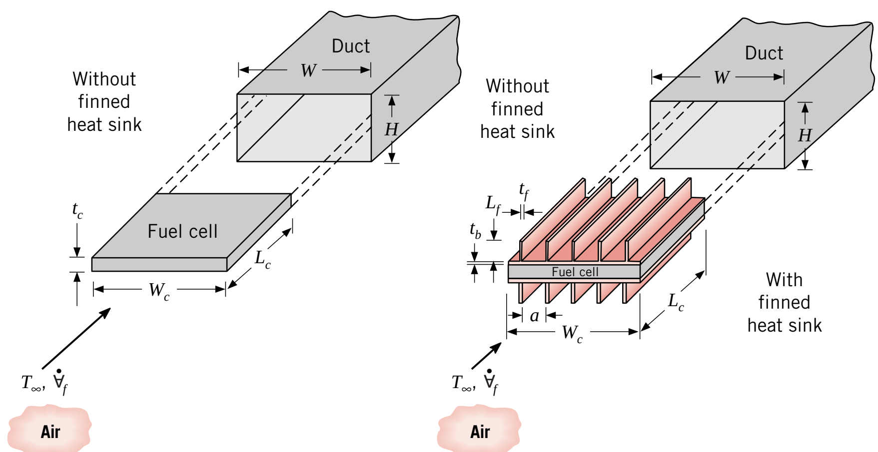


The foregoing calculations are based on the assumption that h is not affected by a reduction in the fin gap. The assumption is reasonable as long as there is no interaction between boundary layers that develop on the opposing surfaces of adjoining fins. Note that, since $NA_f \gg 2\pi r_1(H - Nt)$ for the prescribed conditions, q_t increases nearly linearly with increasing N.

2. The *Models/Extended Surfaces* option of *IHT* provides ready-to-solve models for straight, pin, and circular fins, as well as for fin arrays. The models include the efficiency relations of Figures 3.18 and 3.19 and Table 3.5.

Example 3.11

In Example 1.4, we saw that to generate an electrical power of $P = 9$ W, the temperature of the PEM fuel cell had to be maintained at $T_c \approx 56.4$°C, which required removal of 11.25 W from the fuel cell and a cooling air velocity of $V = 9.4$ m/s for $T_\infty = 25$°C. To provide these convective conditions, the fuel cell is centered in a 50 mm × 26 mm rectangular duct, with 10-mm gaps between the exterior of the 50 mm × 50 mm × 6 mm fuel cell and the top and bottom of the well-insulated duct wall. A small fan, powered by the fuel cell, is used to circulate the cooling air. Inspection of a particular fan vendor's data sheets suggests that the ratio of the fan power consumption to the fan's volumetric flow rate is $P_f/\dot{\forall}_f = C = 1000$ W/(m^3/s) for the range $10^{-4} \le \dot{\forall}_f \le 10^{-2}$ m^3/s.



1. Determine the *net* electric power produced by the fuel cell–fan system, $P_{net} = P - P_f$.
2. Consider the effect of attaching an aluminum ($k = 200$ W/m · K) *finned heat sink*, of identical top and bottom sections, onto the fuel cell body. The contact joint has a thermal resistance of $R''_{t,c} = 10^{-3}$ m^2 · K/W, and the base of the heat sink is of thickness $t_b = 2$ mm. Each of the N rectangular fins is of length $L_f = 8$ mm and thickness $t_f = 1$ mm, and spans the entire length of the fuel cell, $L_c = 50$ mm. With the heat sink in place, radiation losses are negligible and the convective heat transfer coefficient may be related to the size and geometry of a