KING ABDULAZIZ UNIVERSITY

DEPARTMENT OF MATHEMATICS

PhD Entrance Exam

Second Semester 1440

Maximim Time: Three Hours

*Name:*____

Student number ____

Field of specialization ____

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8

Note : Please answer 4 questions of 8.

Name:	
Student Number:	

Q1) Find the equilibrium points, determine their stability and classify their type (node, focus, saddle,...)
(a)

$$
\begin{aligned}
\dot{x} &= y^2 - 3x + 2, \\
\dot{y} &= x^2 - y^2.
\end{aligned}
$$

(b)

$$
\begin{aligned}
\dot{x} &= y, \\
\dot{y} &= \mu(1 - x^2)y - x, \qquad\qquad |\mu| < 2
\end{aligned}
$$

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Q2) **(a)** Find the values of σ so that the following boundary value problem has non-trivial solutions:

$$\begin{aligned} &\frac{d^2u}{dt^2} + (\sigma - 1))u = 0, \\ &u(0) = 0, \ u(1) = 0. \end{aligned}$$

(b) Without solving, find the region through which the solution curve of the following problem passes:

$$\begin{aligned} &\frac{dy}{dx} = (y+1)(y-2), \\ &y(0) = -4. \end{aligned}$$

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Q3) Use Gaussian elimination method to solve the linear system

$$\begin{bmatrix} 4 & 2 & 3 \\ 2 & -4 & -1 \\ -1 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ -5 \end{bmatrix}$$

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Q4) a) Show that the space l^p is complete but not Hilbert space with p$\neq 2$.

b) Show that the dual space of $\mathbb{R}^n$ is $\mathbb{R}^n$.

c) Show that every finite dimensional subspace Y of a normed space X is complete.

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Q5) Let λ denote the Lebesgue measure on $\mathbb{R}$, λ^* denote the Lebesgue outer measure on $\mathbb{R}$. Let $\mathcal{M}$ denote the $\sigma-$ algebra of Lebesgue measurable subsets of $\mathbb{R}$. Let E be subset of $\mathbb{R}$. **You may use the fact that if $E \in \mathcal{M}$, then for every $\epsilon > 0$, there is an open set $O \supseteq E$ such that $\lambda(O - E) < \epsilon$.**

(a) If $E \in \mathcal{M}$. Show that for each $\epsilon > 0$, there is a closed set F with $F \subset E$ and $\lambda(E - F) < \epsilon$.

(b) Suppose that for each $\epsilon > 0$, there is an open set O with $E \subset O$ and $\lambda^*(O - E) < \epsilon$. Prove that $E \in \mathcal{M}$.

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Q6) (a) For any complex number z, show that $z\bar{z} = |z|^2$.

(b) Find the real and imaginary parts of the complex number $\dfrac{5i}{2+i}$.

(c) State the de Moivre's Formula.

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Q7) (a) Prove or disprove: Every group of order 99 is solvable.
(b) Show that every maximal ideal in a commutative ring with unity is a prime ideal.

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Q8) Consider the set of real numbers $\mathbb{R}$. Define $\mathcal{T} \subset \mathcal{P}(\mathbb{R})$ as follows:

$$\mathcal{T} = \{\emptyset, \mathbb{R}\} \cup \{(-x, x) : x \in \mathbb{R}; x > 0\}.$$

(a) Prove that $\mathcal{T}$ is a topology on $\mathbb{R}$.

(b) Prove that $(\mathbb{R}, \mathcal{T})$ is not T_1

(c) Prove that $(\mathbb{R}, \mathcal{T})$ is normal.